



# The dynamical approach to inequality

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Modelling change in socio-economic systems  
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Focus on statistical measurement of inequality from data

Collapse distributions to rankable scalars (Gini, MLD, Theil)

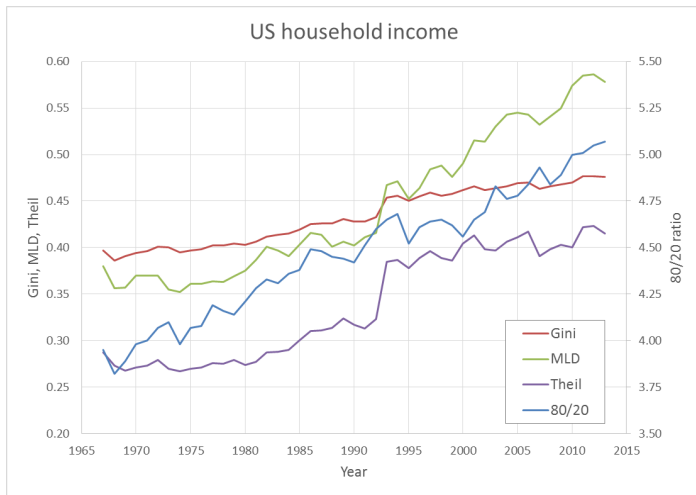
→ “distribution  $A$  is more unequal than distribution  $B$ ”

Empirical distribution as basic epistemological unit

No mechanism for genesis and evolution of distributions



Starting point:



(Source: US Census Bureau)

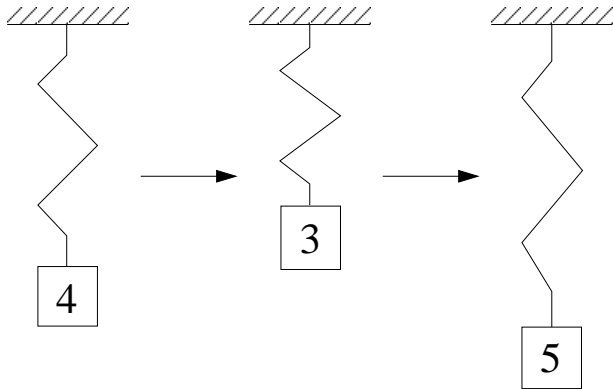


Fundamental question: *how do changes in economies occur?*

Classical economics  $\rightarrow$  equilibrium

Alternative  $\rightarrow$  far-from-equilibrium

Very different implications . . .





*Laissez-faire* policies  $\rightarrow$  stable distributions

Changes track slow evolution of economic conditions

Contradicted by observed instability

Paradigm must appeal to *deus ex machina* explanations,  
e.g. Piketty's *rentier* class (2014)



In equilibrium picture, change is a second order effect

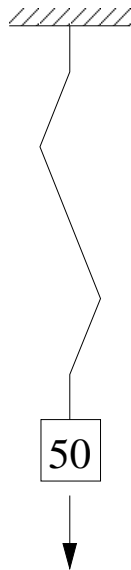
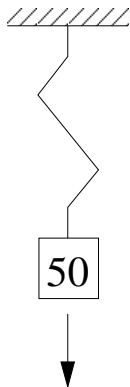
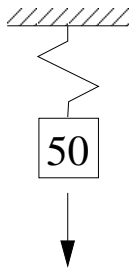
But economies are growing, far-from-equilibrium systems

Classical economics is in cognitive dissonance

Can't explain phenomena arising from dynamical processes



## Far-from-equilibrium







Broadening distributions and rising inequality are defaults

No appeal to speculative special circumstances

*Laissez-faire* policies  $\rightarrow$  wealth condensation

Observed/desired stability requires explanation/intervention



Try to capture salient features of underlying dynamics

Assume individual wealths follow a common stochastic process

→ basic epistemological unit

What is the wealth distribution? How does it evolve?

Find growth rates of *average* and *typical* wealths



Simple, plausible model is noisy multiplicative growth

Continuous time, CLT  $\rightarrow$  geometric Brownian motion

$$dx = x(\mu dt + \sigma dW)$$

Mapping  $u(x) = \ln x$  has stationary independent increments

$$d \ln x = (\mu - \sigma^2/2) dt + \sigma dW$$

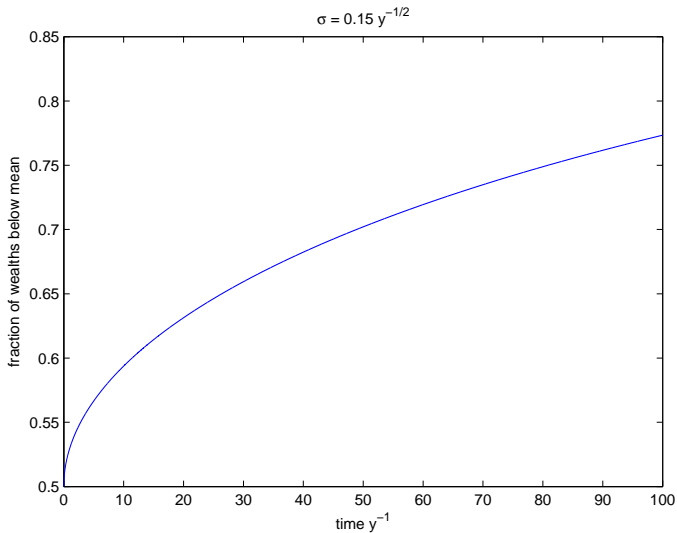


Continually broadening lognormal

$$\ln x(t) \sim \ln x(0) + \mathcal{N}((\mu - \sigma^2/2)t, \sigma^2 t)$$

Get wealth condensation, e.g. fraction of wealths below mean

$$\Pr(x < \langle x \rangle) = \Phi\left(\frac{\sigma\sqrt{t}}{2}\right) \rightarrow 1 \quad \text{as } t \rightarrow \infty$$





Two fundamental growth rates in population

Growth rate of average wealth

$$g_{\text{ave}}(t) = \frac{\Delta \ln \langle x(t) \rangle}{\Delta t} = \mu$$

Growth rate of typical wealth

$$g_{\text{typ}}(t) = \lim_{\Delta t \rightarrow \infty} \left\{ \frac{\Delta \ln x(t)}{\Delta t} \right\} = \frac{\langle \Delta \ln x(t) \rangle}{\Delta t} = \mu - \frac{\sigma^2}{2}$$

(average growth rate of wealth)



Growth rates reflect different economic realities

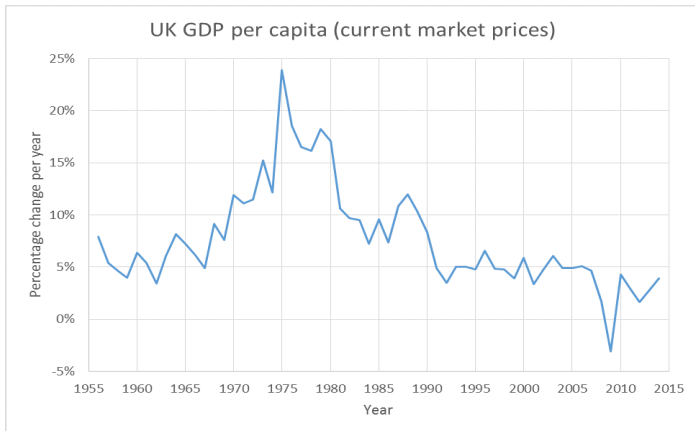
$g_{ave}$   $\longrightarrow$  growth of aggregate quantities, e.g. GDP

$g_{typ}$   $\longrightarrow$  growth as experienced by typical individual

More cognitive dissonance . . .



Gave



(Source: UK Office of National Statistics)





gtyp



(Source: Google images)



# Fundamental theorem of inequality

Central insight: *inequality changes when there is a difference between the two growth rates*

*Define inequality as quantity which grows at difference of rates*

$$\frac{dJ}{dt} = g_{\text{ave}}(t) - g_{\text{typ}}(t)$$

Multiplicative growth:  $J(t) = \ln \langle x(t) \rangle - \langle \ln x(t) \rangle = \text{MLD}$

→ dynamical interpretation of widely-used measure

Geometric Brownian motion:  $J(t) = \sigma^2 t/2$



Different wealth dynamics  $\longrightarrow$  different  $u(x)$

- Choice of inequality metric becomes choice of dynamic
- Debates can be settled empirically

Include taxation and redistribution to stabilise distribution

- Intervention framed mechanistically *via* dynamic
- Critical tax rate emerges as stabilisation criterion



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